Corollary. *For every graph G the following hold:*

- *There is always a vertex of degree at least* $\lfloor \frac{2m}{n} \rfloor$.
- *There is always a vertex of degree at most* $\lceil \frac{2m}{n} \rceil$.

There's always more to say...

Theorem (Erdős-Gallai)**.** *Let* $d_1 \geq d_2 \geq ... \geq d_n$ *be a sequence of integers.*

1. *If we allow loops and parallel edges, then there exists a graph with degree sequence* $(d_1, d_2, ..., d_n)$ *if and only if* $d_1 + d_2 + ... + d_n$ *is even.*
2. *If we allow parallel edges, then there exists a graph with degree sequence* $(d_1, ..., d_n)$ *if and only if* $d_1 + ... + d_n$ *is even and* $d_1 \leq d_2 + ... + d_n$.
3. *There exists a graph with degree sequence* $(d_1, ..., d_n)$ *if and only if* $d_1 + ... + d_n$ *is even and for each* $1 \leq k \leq n$ *we have that* $d_1 + ... + d_k \leq k(k-1) + min(d_{k+1}, k) + ... + min(d_n, k)$.